HALL EFFECTS ON MHD FLOW IN A ROTATING SYSTEM WITH MOVING HORIZONTAL POROUS PLATE

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Abstract

In this paper,the solution of the problem of Hall current on MHD flow in a rotating system with an accelerated horizontal porous plate has been presented.The dimensionless governing equations of the flow problem are solved by Pertubation technique.A uniform magnetic field is assumed to be applied transversely to the direction of the flow.The expressions for velocity fields and skin friction are obtained in non-dimensional form.The effects of the different parameters namely,rotational parameter,Hartmann number,Hall parameter and acceleration parameter are discussed.

Keywords: *MHD; Rotating System; Transient; Hall Current; Mass Transfer; Skin-Friction*

Introduction

Many engineering problems are susceptible to MHD analysis. The transversely applied magnetic field has immediate applications in many devices such as magneto hydro dynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, and polymer technology, and petroleum industry, purification of crude oil and fluid droplets sprays. Geophysics encounters MHD phenomena in interactions of conducting fluids and magnetic fields. The rotating flow of an electrically conducting fluid in presence of magnetic field has got its importance in Geophysical problems. The study of rotating flow problems are also important in the solar physics dealing with the sunspot development, the solar cycle and the structure of rotating magnetic stars. It is well known that a number of astronomical bodies possess fluid interiors and magnetic fields. Changes that take place in the rate of rotation, suggest the possible importance of hydro magnetic spin up .The general theory of rotating fluids has received growing interest during last decade because of its application in Cosmic and Geophysical science.

In this regard, we may cite the works done by Raptis [1], Singh [2], MHD in the present form is due to the pioneer contribution of several notable authors like Alfven [3], Ferraro and etc. It was emphasized by Cowling (1975) that when the strength of the applied magnetic field is sufficiently large, Ohm's law needs to be modified to include Hall current. The Hall effect is due merely to the sideways magnetic force on the drifting free charges. The electric field has to have a component transverse to the direction of the current density to balance this force. In many works of Plasma physics, it is not paid much attention to the effect caused due to Hall current. However, the Hall effect cannot be completely ignored if the strength of the magnetic field is high and the number density of electrons is small as it is responsible for the change of the flow pattern of an ionized gas .Hall effect results in a development of an additional potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. This current is termed as Hall current. It

was discovered in 1879 by Edwin Herbert Hall while working on his doctoral degree at the Johns Hopkins University in Baltimore, Maryland, USA. Pop [4], Kinyanjui *et al*. [5], Archrya *et al*. [6] .etc. have presented some model studies on the effect of Hall current on MHD convection flow because of its possible application in the problems of MHD generators and Hall accelerators. An unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat transfer have been studied by Ahmed *et al*. [7]. Recently, Ahmed and Sarmah [8] have carried out an investigation of MHD transient flow past an impulsively started infinite horizontal porous plate in a rotating system with Hall current .Due to the importance of studying MHD flow problems in rotating fluid with Hall current, we have proposed in the present paper to investigate an unsteady
MHD transient flow with Hall currents past a uniformly accelerated porous plate in a rotating system.

Basic Equations

The governing equations are

Equation of continuity:

$$\nabla \cdot q = 0 \tag{1}$$

Momentum equation:

$$\circ\left[{\partial q}/{\partial t} + 2\Omega\times q + \Omega\times(\Omega\times r) + (q.\nabla)q\right] = -\nabla p + J\times B + «\nabla^2 q \tag{2}$$

Kirchhoff's first law:

$$\nabla . J = 0 \tag{3}$$

General Ohm's law:

$$J + \frac{\mu^2}{B_0}(J\times B) = \pm\left[E + q\times B + \frac{1}{e¦}\nabla p\right] \tag{4}$$

Gauss's law of magnetism:

$$\nabla . B = 0 \tag{5}$$

where ***q*** is the velocity vector,

Ω Angular velocity of the fluid,

r the position vector of the fluid particle *P*,

P the pressure,

J the current density,

B the magnetic induction vector,

μ the co-efficient of viscosity,

σ the electrical conductivity,

t′ the time,

B₀ the strength of the applied magnetic field,

ω the electron frequency,

τ the electron collision time,
e the electron charge,
η the number density of electron,
p the electron pressure,
E the electric field,
2Ω x ***q*** is the Coriolis acceleration,
Ω x (Ω x ***r)*** is the centripetal acceleration and the other symbols have their usual meanings and the other symbols have their usual meanings

We now consider an unsteady flow of an incompressible viscous electrically conducting fluid past a suddenly started infinite horizontal porous plate relative to a rotating system with constant suction in presence of a uniform transverse magnetic field taking into account the effect of Hall current. Our investigation is restricted to the following assumptions:

- All the fluid properties are constants and the buoyancy force has no effect on the flow.
- The plate is electrically non-conducting.
- The entire system is rotating with angular velocity Ω about the normal to the plate and IΩ I is so small that IΩ x (Ω x ***r)*** Ican be neglected.
- The magnetic Reynolds number is so small that the induced magnetic field can be neglected.

Initially the plate and the fluid were rotating in unison with a constant angular velocity Ω about the normal to the plate. At time t`>0 , the plate is moved in its own plane relative to the rotating system with acceleration $a$`.

We introduce a coordinate system (x',y', z')with X-axis horizontally in the direction of the plate velocity, Y-axis horizontally perpendicular to the direction of the plate velocity and Z-axis along the normal to the plate which is the axis of rotation. Let ***q*** $=\hat{i}u'+ \hat{j}v'+ \hat{k}w'$ be the fluid velocity ***J*** $=J_x$ i $+J_y j +J_z k$ be the current density at the point P (x', y', z' ,t') and ***B*** $=B_0 \hat{k}$ be the applied magnetic field, $\hat{i}$, $\hat{j}$, $\hat{k}$ being the unit vectors along X-axis, Y-axis and Z-axis respectively. As the plate is infinite in X-direction and Y-direction, therefore all the quantities except possibly the pressure are independent of x' and y'.

Using Boundary layer approximation the basic equation reduces to

$$\frac{\partial u}{\partial t}-w_0\frac{\partial u}{\partial z}-2\Omega v=v\frac{\partial^2 u}{\partial z^2}+\frac{\pm B_0^2\left[mv-u)\right]}{\circ\left(1+m^2\right)} \tag{6}$$

$$\frac{\partial v}{\partial t}-w_0\frac{\partial v}{\partial z}-2\Omega u=\,^3\,\frac{\partial^2 v}{\partial z^2}+\frac{\pm B_0^2\left[mu+v)\right]}{\circ\left(1+m^2\right)} \tag{7}$$

With $\frac{\partial p}{\partial z}=0$

Subject to the initial and boundary conditions:

$$u=0, v=0 \ for\, t\le o, \forall z \tag{8}$$

$$\left.\begin{array}{l} u = at, v = 0 \ at \ z = o \\ u = 0, v = 0 \, at \, z \to \infty \end{array}\right\} \forall t > 0$$

Method of Solution

The non-dimensional forms of the equation governing the flow can be rewritten as follows: Let us introduce the complex form of q defined by q = u +iv.

Equation (6) & (7) redues to

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + \frac{\partial q}{\partial z} - Aq \qquad (9)$$

Where $A = i\Omega + \frac{M(1+im)}{1+m^2}$

Subject to the boundary conditions :

$$q = 0 \ for \, t \le 0 \qquad (10)$$

$$\left.\begin{array}{l} q = at, v = 0 \ at \ z = 0 \\ q = 0, v = 0 \ at \ z \to \infty \end{array}\right\} \forall t \qquad (11)$$

Let us assume the following equation:

$$q(z,t) = q_0(z) + \pounds q_1(z) e^{-\mu t} + ... \qquad (12)$$

By applying the above assumption to the governing we get the corresponding equations as,

Zeroth Order

$$q_0''(z) + q_0'(z) - Aq_0 = 0 \qquad (13)$$

First Order

$$q_1''(z) + q_1'(z) - (w + A) q_1(z) = 0 \qquad (14)$$

Solving the Equations we Get the Velocity Profile as

$$q(z,t) = (A_1 e^{m_1 z} + B_1 e^{m_2 z}) + \pounds e^{-\mu t} (A_2 e^{m_3 z} + B_2 e^{m_4 z}) \qquad (15)$$

Conclusion

- The main flow velocity u decreases with an imposition of magnetic field.
- An increase in accelerating parameter a results in a remarkable growth in the main flow velocity u in a thin layer adjacent to the plate.

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